Nonlinear Structural Coupling - Experimental Application

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ABSTRACT

In this work, the nonlinear structural modification/coupling technique proposed recently by the authors is applied to a test system in order to study the applicability of the method to real structures. The technique is based on calculating the frequency response functions of a modified system from those of the original system and the dynamic stiffness matrix of the nonlinear modifying part. The modification can also be in the form of coupling a nonlinear system to the original system. The test system used in this study is composed of two cantilever beams with their free ends held between two thin identical beams which yield cubic stiffness. Thus two linear structures are coupled with a nonlinear connecting element. The frequency response functions of the coupled nonlinear system are measured at several different harmonic forcing levels, and the experimental values are compared with the theoretical values calculated using the nonlinear structural coupling method.

Keywords: *Nonlinear structural coupling, nonlinear coupling, structural coupling analysis, nonlinear vibrations, substructure synthesis.*

1 INTRODUCTION

Engineering structures are generally very complex and it is difficult to analyze them for their dynamic behavior. Structural modification and coupling methods can be used as a shortcut in predicting the dynamic behavior of a modified/coupled structure instead of reanalyzing it as a whole. Structural modification/coupling methods are standard tools in structural dynamics for linear structures. Many different methods have been employed for modified/coupled linear structures [1-12].

In last two decades, structural coupling methods taking the effects of nonlinear connection elements into account have been suggested. Watanabe and Sato [13] proposed the so-called Nonlinear Building Block approach to obtain the frequency response of a structural system with nonlinear springs connecting linear components. They used the spatial coupling method and represented the joint nonlinearity with its describing function. Wyckaert et al. [14] proposed a method which predicts the response of linear structures coupled with nonlinear joints, using describing function approach. Cömert and Özgüven [15] developed a method for calculating the forced response of linear substructures coupled with nonlinear connecting elements. They used frequency response functions (FRFs) of the linear substructures and properties of the nonlinear connection elements in order to obtain forced harmonic response of the coupled structure. Ferreira and Ewins [16] proposed a new method for fundamental harmonic analysis of nonlinear system using describing functions. This method is capable of coupling structures with local nonlinear elements whose describing functions are available, considering only the harmonic motion at fundamental frequency. Ferreira [17] extended this approach and introduced Multi-Harmonic Nonlinear Receptance Coupling method. This method is capable of coupling linear structures with different types of nonlinear joints by substituting internal nonlinear forces by their corresponding multi-harmonic describing functions. Huang [18] also proposed a method for predicting dynamic responses of a complex structural assembly with nonlinearity at joints.

In this paper, applicability and validity of the nonlinear structural modification/coupling method proposed recently by the authors [19] are demonstrated by applying it on a test structure and by comparing the experimentally measured nonlinear FRFs with those calculated using the method. The method proposed is for structural modification analysis problems where a linear system can be modified with a nonlinear subsystem for the following three cases: structural modification with additional degrees of freedom (DOFs), structural coupling with linear elements and structural coupling with nonlinear elements. In the previous work, the applicability of the method is demonstrated for all those cases by using several theoretical case studies. In this work, the proposed approach is applied to an experimental test system in which two linear structures are coupled by a nonlinear element.

2 THEORY

The structural modification method proposed by Özgüven [12] has recently been extended to nonlinear systems [19, 20]. The formulation was given for two different cases: structural modification without adding new DOFs to the original system, and structural modification which increases the total DOF of the original system (this formulation is also applicable to structural coupling problems). The resultant equations for the second case are given as follows [12]:

$$\begin{bmatrix} [H_{ba}^{*}] \\ [H_{ca}^{*}] \end{bmatrix} = \left[\begin{bmatrix} [I] & [0] \\ [0] & [0] \end{bmatrix} + \begin{bmatrix} [H_{bb}] & [0] \\ [0] & [I] \end{bmatrix} . [Z] \right]^{-1} \begin{bmatrix} [H_{ba}] \\ [0] \end{bmatrix} \tag{1}$$

$$\begin{bmatrix} [H_{bb}^{*}] & [H_{bc}^{*}] \\ [H_{cb}^{*}] & [H_{cc}^{*}] \end{bmatrix} = \left[\begin{bmatrix} [I] & [0] \\ [0] & [0] \end{bmatrix} + \begin{bmatrix} [H_{bb}] & 0 \\ 0 & [I] \end{bmatrix} . [Z] \right]^{-1} \begin{bmatrix} [H_{bb}] & [0] \\ [0] & [I] \end{bmatrix} \tag{2}$$

$$[H_{aa}^{*}] = [H_{aa}] - \left[[H_{ab}] \,\vert\, [0] \right] [Z] \begin{bmatrix} [H_{ba}^{*}] \\ [H_{ca}^{*}] \end{bmatrix} \tag{3}$$

$$\left[[H_{ab}^{*}] \,\vert\, [H_{ac}^{*}] \right] = \left[[H_{ab}] \,\vert\, [0] \right] \left[[I] - [Z] \begin{bmatrix} [H_{bb}^{*}] & [H_{bc}^{*}] \\ [H_{cb}^{*}] & [H_{cc}^{*}] \end{bmatrix} \right] \tag{4}$$

where $[H]$ and $[H^*]$ are the receptance matrices of the original and modified systems, respectively. Here, the subscript a represents the coordinates that belong to the original system only, the subscript b denotes connection coordinates, and the subscript c represents coordinates that belong to modifying system only. $[Z]$ denotes the dynamic stiffness matrix of the modifying structure, which can be written for a linear modification case as follows:

$$[Z] = [\Delta K] - \omega^2 [\Delta M] + j\omega [\Delta C] + j [\Delta D] \tag{5}$$

where $[\Delta K]$, $[\Delta M]$, $[\Delta C]$ and $[\Delta D]$ represent stiffness, mass, viscous and structural damping matrices of the modifying structure, respectively. For a nonlinear modification $[Z]$ will be expressed as [19, 20]:

$$[Z(X)] = [\Delta K] - \omega^2 [\Delta M] + j\omega [\Delta C] + j [\Delta D] + [\Delta(X)] \tag{6}$$

where $[\Delta(X)]$ is the "nonlinearity matrix" whose elements are functions of unknown response amplitudes and can be written in terms of describing functions for any type of nonlinearity as [21]:

$$\Delta_{kk} = \sum_{m=1}^{n} v_{km}, \qquad k = 1, 2, \ldots, n \tag{7}$$

$$\Delta_{km} = -v_{km}, \qquad k \neq m, \qquad k = 1, 2, \ldots, n \tag{8}$$

Here, subscripts k and m represent two engagement coordinates, and v is the first order describing function of a nonlinear element in the system. Derivation of v for various types of nonlinearities can be found in several references (for instance, see Refs. [21-23]). As $[\Delta(X)]$ is a function of the unknown response amplitude $\{X\}$, when the method is used for nonlinear modification an iterative solution is required [21].

The above formulation is valid for rigid coupling of a nonlinear system to a linear system as well. However, when the systems are not coupled rigidly but with nonlinear elastic elements, then the nonlinear modifying system can be constructed as shown in Fig. 1. By this way, the problem turns out to be a nonlinear structural modification problem with additional DOFs which can easily be handled by using the approach given above.

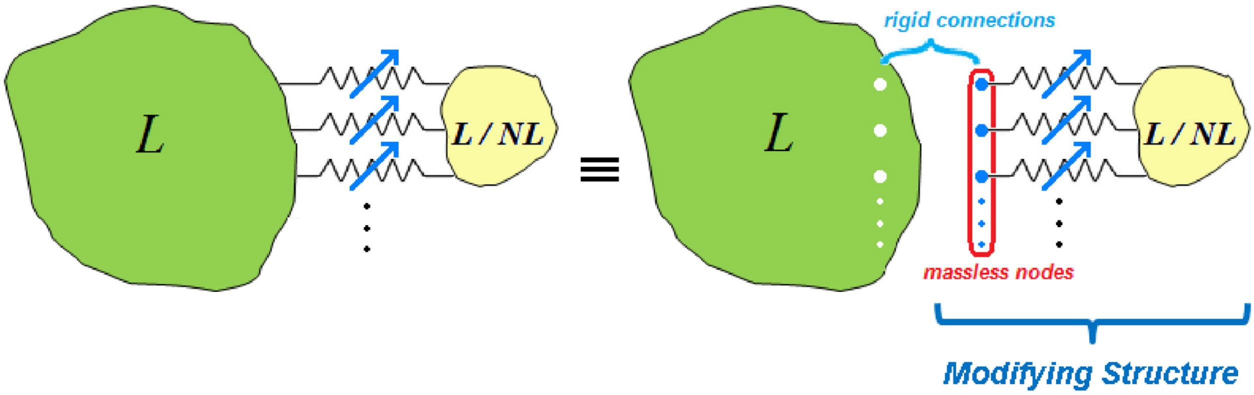

Fig. 1 Structural coupling of two systems with nonlinear elements

3 EXPERIMENTAL STUDY AND VERIFICATION

The nonlinear structural modification/coupling technique proposed is applied to a test system. In this experimental study, a cantilever beam is coupled to a shorter cantilever beam by connecting their free ends to each other with two thin identical beams which yields hardening cubic stiffness. The nonlinear FRFs of the coupled system composed of two linear structures coupled with a nonlinear element are measured at several different harmonic forcing levels, and the experimental values are compared with theoretical values obtained using the nonlinear coupling method developed.

3.1 Experimental setup

The test rig manufactured for this study is shown in Fig. 2. The dimensions and technical details of the experimental setup are given in Fig. 3. The cantilever beams and the thin beams yielding cubic stiffness were manufactured from St37 steel.

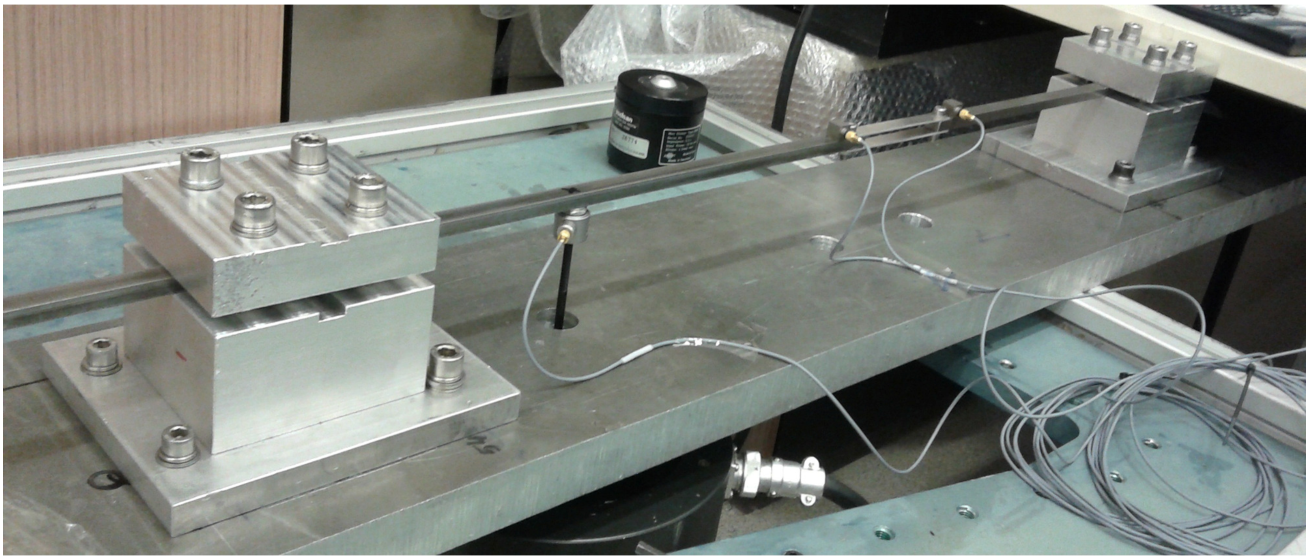

Fig. 2 Setup used in the experimental study

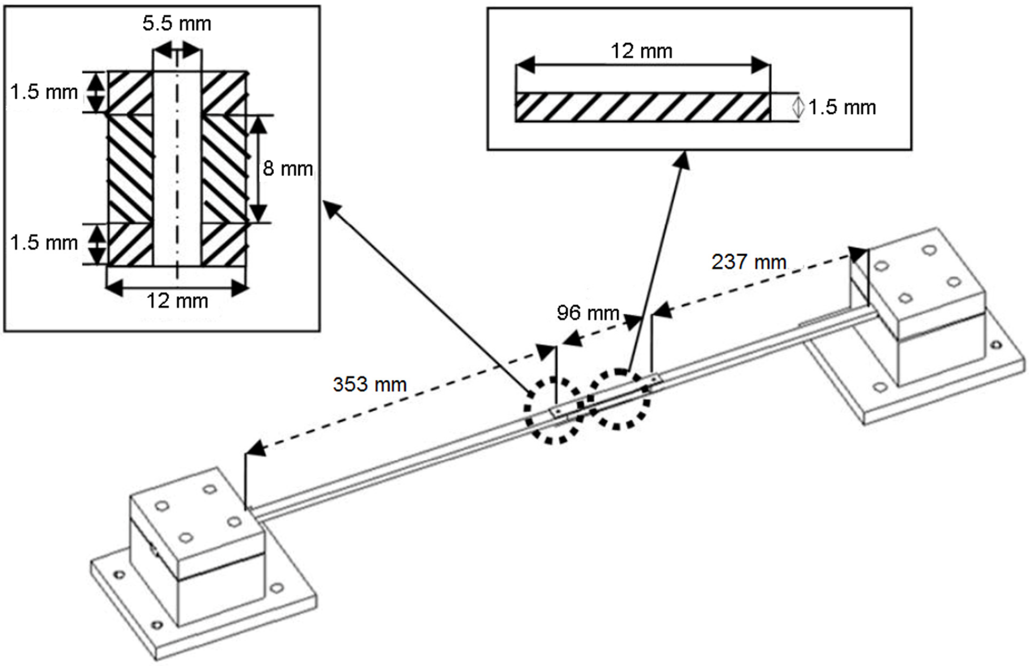

Fig. 3 Dimensions and technical details of the test system

The modal test setup configuration is shown in Fig. 4. For linear and nonlinear testing, a shaker (B&K 4808) was connected to a point 112 mm away from the fixed end of the long cantilever beam via a push-rod with a force transducer (B&K 8230-002). The vibration responses were measured using two accelerometers (B&K 4524) from the point of excitation and from the tip of the long cantilever beam. Since cubic stiffness nonlinearity in the system causes jumps in the nonlinear frequency response around resonance frequencies, the ability to observe this phenomenon is closely related to the frequency resolution employed in the harmonic vibration tests. Therefore the frequency resolution was generally taken as 0.25 Hz in almost entire frequency range of interest, and it is halved where jump occurs.

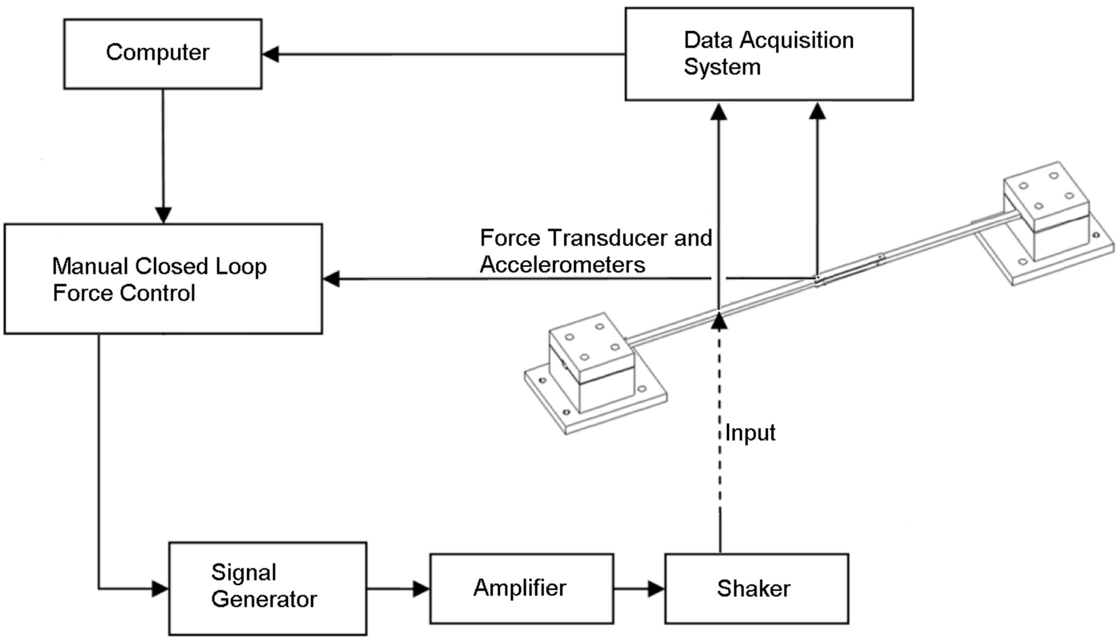

Fig. 4 Experimental setup configuration

The modal tests are performed using harmonic forcing with amplitudes of 0.5 N and 1 N. Note in Fig. 4 that force level is controlled manually for each frequency step.

3.2 Computation of nonlinear FRFs for the test structure

3D elastic beam elements of ANSYS are used in the finite element (FE) model of the cantilever beams. Beam elements have 3 DOFs per node yielding 1062 total DOFs for the long cantilever beam, and 39 total DOFs for the short cantilever beam (Fig. 5).

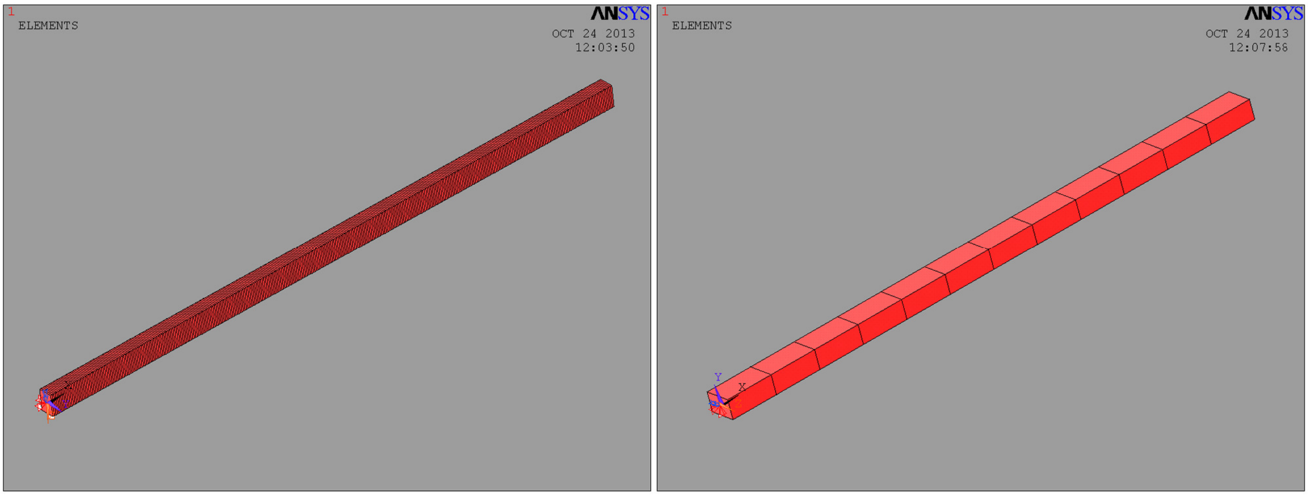

Fig. 5 The FE models of the long and the short cantilever beams

Note here that the long cantilever beam is intentionally fine-meshed in order to show that the computational time of the proposed approach is not affected by the DOF of the original system. Because, the proposed approach is FRF based, and for the original structure, only the FRFs of the desired DOFs, in addition to those of connection DOFs, are included in the calculations.

Structural damping ratio of the system is identified from the linear FRFs (FRFs measured with low forcing) as 0.009. The other material properties of all the beams used in the experiment, which are made of St37 steel, are given in Table 1.

Table 1. Material properties of the beams

Young's Modulus	210 GPa
Poisson's Ratio	0.30
Density	7850 kg/m^3

Using these FE models, FRFs of the long cantilever beam (original structure) and system matrices of the short cantilever beam (modifying/coupled structure) are obtained using ANSYS. In order to obtain the equivalent linear and nonlinear flexural stiffnesses introduced by the thin beams, linear and nonlinear large deflection analyses are performed in ANSYS. Then by fitting a polynomial curve composed of linear and cubic terms only to the force-deflection data, the equivalent linear and nonlinear flexural stiffness values are obtained as 7600 N/m and 3×10^9 N/m^3, respectively.

3.3 Comparison of calculated and measured nonlinear FRFs

The nonlinear transfer FRFs measured from tip of the long cantilever beam by performing constant amplitude force tests are compared with those obtained via the proposed approach in Fig. 6 and Fig. 7 for each forcing level. Test is carried out by applying pure sine signal of different voltages to keep the forcing level constant at each frequency step.

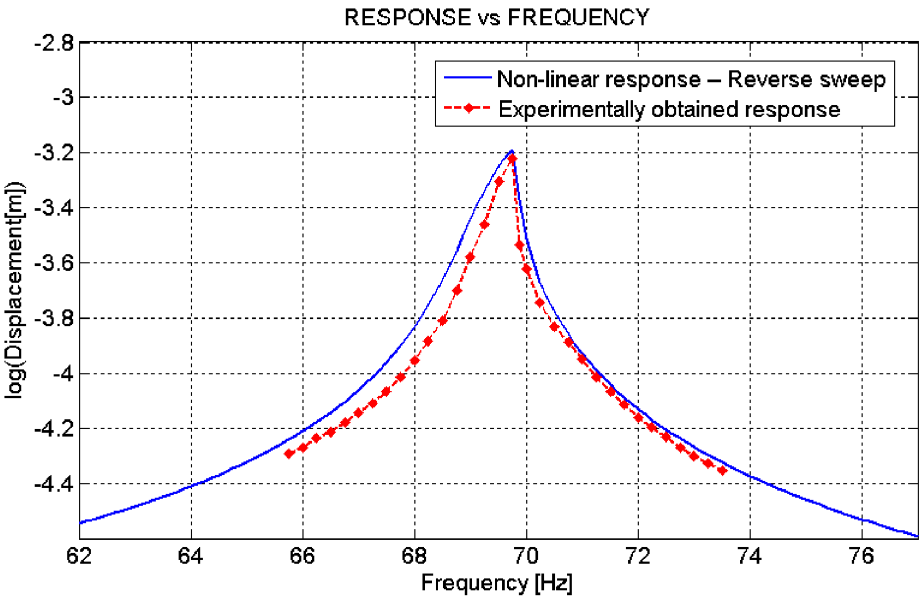

Fig. 6 Calculated and measured nonlinear transfer FRFs at 0.5 N force amplitude level

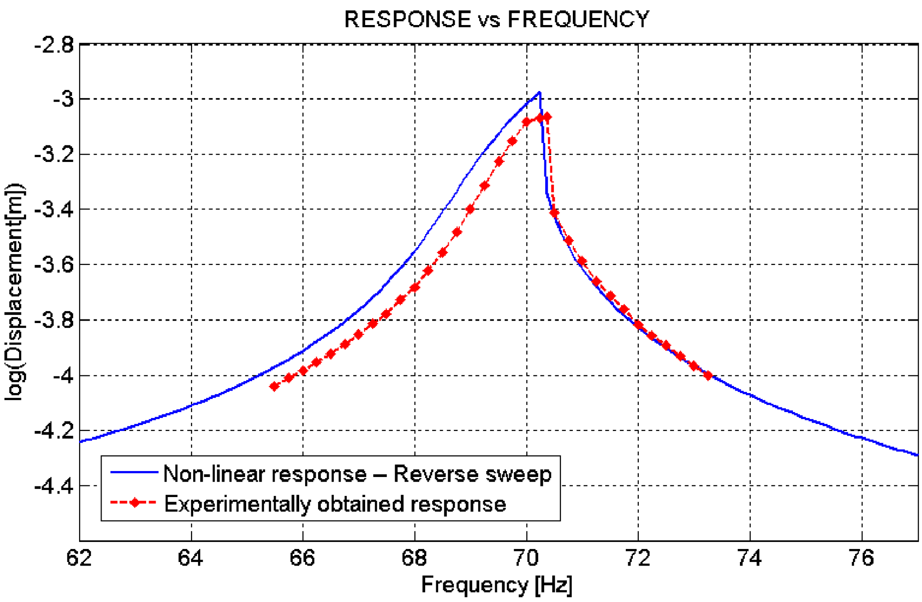

Fig. 7 Calculated and measured nonlinear transfer FRFs at 1 N force amplitude level

As can be seen in Fig. 6 and Fig. 7, fairly good agreements are obtained between experimental and predicted nonlinear FRF values even at high response levels where jump occurs in the frequency response. Differences are believed to be due the system parameters used in the model and partly due to pure manual control of forcing level during tests.

4 DISCUSSION AND CONCLUSIONS

The structural modification method developed for linear systems almost two decades ago [12] was previously extended for dynamic reanalysis of linear structures modified locally with a nonlinear substructure, as well as for dynamic reanalysis of linear structures coupled with nonlinear substructures by using linear and/or nonlinear coupling elements [19, 20]. In this study another possible implementation of the method, that is dynamic reanalysis of linear structures coupled with nonlinear elements, is demonstrated.

The approach is based on the computation of the nonlinear FRFs of a modified system from linear FRFs of the original system and the dynamic stiffness matrix representing the nonlinear modifications in the system. Due to the nonlinear behavior of the modifying system, the dynamic stiffness matrix representing the modifications turns out to be response level dependent, and therefore the solution requires an iterative approach. For the case given in this study, construction of the nonlinear modifying system is achieved through incorporation of nonlinear connection elements into linear coupling substructure, such that to each free end of a connecting nonlinear element a massless node is added. Then the nonlinear substructure is rigidly connected to the main system.

The validity and performance of the proposed technique for the above case is demonstrated through an experimental study. The nonlinear transfer FRFs calculated using the proposed approach are compared with those obtained experimentally. An acceptable agreement is observed between the predicted and measured results even though nonlinear parameters of two thin beams are obtained approximately via curve fitting to the force-displacement data obtained from large deflection analysis in a FE program. Furthermore, it is believed that keeping the forcing level constant in each test at different frequencies by only adjusting the shaker voltage manually has also introduced some error into the experimental results.

The method used is an FRF based method, therefore only the FRFs of the original system for the desired DOFs, in addition to those of connection DOFs, are included in the calculations, which reduces the computational time drastically, especially for large ordered systems. That is, as long as the modifying subsystem is of small order, the computational cost does not increase with increasing size of the original system. For such cases, the method is very useful and makes it possible for the designer to try various possible design changes or to make a parametric study with minimum computational cost. The test system used here is very typical in that sense; for example in investigating the effects of different nonlinear connecting elements on modified system response can very easily be studied in this problem.

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